PROPAGATION OF NONLINEAR PERTURBATION WAVES IN RHEOLOGICALLY COMPLEX LIQUIDS

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The propagation of perturbation waves in real liquids moving in elastic and elastoviscous tubes has been considered several times in the literature [1-5]. Those studies examined the effects of rheological characteristics on perturbation wave propagation in tubes with Newtonian and non-Newtonian (viscoplastic, power-like, relaxation) liquids. The problems considered essentially reduced to solution of linearized systems of equations describing the Newtonian and non-Newtonian liquids. However, the linearization used was not always justified, since consideration of nonlinearity may lead to qualitatively new effects. Thus the present study will investigate propagation of nonlinear waves in rheologically complex liquids in motion within tubes.

The system of differential equations describing the motion of a real liquid in tubes has the form [1]

$$\partial(f\rho w)/\partial t + \partial(f\rho w^2)/\partial x = -f\partial p/\partial x + \chi\tau(w),\ \partial(f\rho)/\partial t + \partial(f\rho w)/\partial x = 0, \tag{1}$$

where f is the cross-sectional area of the tube, ρ is the liquid density, w is the mean liquid velocity over tube section, p is pressure, χ is the wetted perimeter, τ is the tangent stress, x is the direction of the flow, and t is time.

We assume that the cross-sectional area of the tube depends on pressure in accordance with Hook's law:

$$f = f_0(1 + (p - p_0)/E), \tag{2}$$

while the equation of state of the non-Newtonian liquid has the form

$$p - p_0 = c_0^2\left(\rho_1 + \frac{\gamma-1}{2\rho_0}\rho_1^2\right), \tag{3}$$

where f_0 is the tube area at pressure p_0; α is some dimensionless coefficient depending on the form of the cross section and the wall thickness; E is the modulus of elasticity of the tube material; γ is a constant characterizing the medium; $c_0 = (dp/d\rho)^{1/2}$ is the unperturbed wave velocity; ρ_0 is the unperturbed density; ρ_1 is the change in density.

With consideration of Eqs. (2), (3), after several transformation system (1) may be written in the form

$$\frac{\partial w}{\partial t} + \frac{c_0^2}{\rho_0}\frac{\partial \rho_1}{\partial x} = -\frac{c_0^2(\gamma-1)}{2\rho_0}\frac{\partial \rho_1^2}{\partial x} - \frac{1}{2}\frac{\partial w^2}{\partial x} + \frac{\chi}{\rho_0 f_0}\tau(w),$$
$$\left(\frac{a}{E}c_0^2 + \frac{1}{\rho_0}\right)\frac{\partial\rho_1}{\partial t} + \frac{a}{E}\frac{c_0^2}{2\rho_0}(\gamma+1)\frac{\partial\rho_1^2}{\partial t} + \frac{\partial w}{\partial x} = -\left(\frac{a}{E}c_0^2 + \frac{1}{\rho_0}\right)\frac{\partial\rho_1 w}{\partial x} - \frac{c_0^2}{2\rho_0}(\gamma+1)\frac{\partial\rho_1^2 w}{\partial x}. \tag{4}$$

For a specified friction law the system of nonlinear equations can be solved by numerical methods. An approximate solution of system (4) can be obtained by using certain simplifications [6].

After a number of transformations and use of approximations ($\partial/\partial t \simeq -c_1\partial/\partial x$, $w \simeq c_1\rho_1/\rho_{00}$, third-order density value neglected, and operator $\partial/\partial x$ omitted) system (4) can be written in the form

$$\frac{\partial\rho_1}{\partial t} + (c_1 + 2\alpha\rho_1)\frac{\partial\rho_1}{\partial x} = \beta\tau(\rho_1), \tag{5}$$

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where

$$c_1 = \frac{c_0}{\left(a\frac{c_0^2}{c_T^2}+1\right)^{\frac{1}{2}}}, \quad \rho_{00} = \frac{\rho_0}{a\frac{c_0^2}{c_T^2}+1},$$

$$\alpha = \left[\frac{3}{2}\frac{c_0^2}{\rho_0} + \frac{c_0(\gamma-1)}{2\rho_0\left(a\frac{c_0^2}{c_T^2}+1\right)} - \frac{ac_0^4(\gamma+1)}{2\rho_0 c_T^2\left(a\frac{c_0^2}{c_T^2}+1\right)^2}\right]\frac{\left(a\frac{c_0^2}{c_T^2}+1\right)^{\frac{1}{2}}}{2c_0},$$

$$\beta = 1\Bigg/\left(2c_0\delta\left(a\frac{c_0^2}{c_T^2}+1\right)^{\frac{1}{2}}\right), \quad \delta = \frac{f_0}{\chi}.$$

The friction law $\tau(\rho_1)$ can be written easily if the rheological curve of non-Newtonian liquid flow $\tau(w)$ is specified.

A characteristic feature of the solution of hyperbolic equation (5) when dissipation is neglected in the liquid ($\tau(\rho_1) \equiv 0$) is the distortion of the propagating wave profile due to nonlinearity in the wave velocity [7]. Consideration of dissipation in the liquid naturally leads to attenuation of the wave, while features of the friction law may have a significant effect on the attenuation process.

We will consider propagation of perturbation waves in non-Newtonian liquids described by the following rheological equations of state:

$$\tau(w) = -k_0 w^n \tag{6}$$

for a power-law liquid;

$$\theta\frac{d\tau}{dt} + \tau = -\frac{8\mu}{D}\left(w + \lambda\frac{dw}{dt}\right) \tag{7}$$

for an Oldroyd viscoelastic (relaxation) liquid.

The choice of the model of Eq. (7) is based on studies performed in [8, 9] with petroleums containing paraffin, resin, and asphalt components.

With consideration of the approximate equality used above, Eqs. (6) and (7) may be represented in the forms

$$\tau(\rho_1) = -k\rho_1^n; \tag{8}$$

$$\theta d\tau/dt + \tau = -k_1(\rho_1 + \lambda d\rho_1/dt), \tag{9}$$

where $k = k_0(c_1/\rho_{00})^n$; $k_1 = 8\mu c_1/D\rho_{00}$; k_0, n are the dimensionless rheological parameters of the consistent flow curve; μ is the liquid viscosity; D is the tube diameter; θ, λ are the relaxation and retardation times.

The solution of Eq. (9) for τ has the form

$$\tau = \left[\tau|_{t=0} - k_1\frac{\lambda}{\theta}\rho_1|_{t=0}\right]e^{-\frac{t}{\theta}} + k_1\left(1-\frac{\lambda}{\theta}\right)e^{-\frac{t}{\theta}}\rho_1|_{t=0} - \\ - k_1(\theta-\lambda)e^{-\frac{t}{\theta}}\frac{d\rho_1}{dt}\bigg|_{t=0} - k_1\rho_1 + k_1(\theta-\lambda)\frac{d\rho_1}{dt} - k_1(\theta-\lambda)\int_0^t e^{-\frac{t-t_1}{\theta}}\frac{d^2\rho_1}{dt_1^2}dt_1. \tag{10}$$

We assume that at the initial moment $t = 0$, $x = \xi$ some perturbation is specified: $\rho_1 = \rho_{11}(\xi)$ (for a power-law liquid) and $\tau|_{t=0} = k_1\lambda\rho_1/\theta|_{t=0}$, $\rho_1|_{t=0} = \rho_{11}(\xi)$, $d\rho_1/dt|_{t=0} = 0$ (for an Oldroyd viscoelastic liquid).

The solution of Eq. (5) with consideration of Eqs. (8), (10) reduces to integration of ordinary differential equations along characteristics [7]:

$$\frac{d\rho_1}{dt} = -\beta k\rho_1^n, \quad \frac{dx}{dt} = c_1 + 2\alpha\rho_1; \tag{11}$$

$$[1-\beta k_1(\theta-\lambda)]\frac{d\rho_1}{dt}=\beta k_1\left(1-\frac{\lambda}{\theta}\right)e^{-\frac{t}{\theta}}\rho_{11}(\xi)-\beta k_1\rho_1-\beta k_1(\theta-\lambda)\int_0^t e^{-\frac{t-t_1}{\theta}}\frac{d^2\rho_1}{dt_1^2}dt_1, \tag{12}$$

$$\frac{dx}{dt}=\frac{c_1}{1-\beta k_1(\theta-\lambda)}+\frac{2\alpha-\beta k_1(\theta-\lambda)\frac{c_1}{\rho_{00}}}{1-\beta k_1(\theta-\lambda)}\rho_1,$$

where

$$\beta k_1(\theta-\lambda)>1.$$

With consideration of initial conditions the solution of system (11), (12) has the following form:
for a power-law liquid

$$\rho_{12}=\frac{\rho_1}{\rho_{11}\xi}=\left\{1-\frac{(1-n)\beta k}{[\rho_{11}(\xi)]^{1-n}}t\right\}^{\frac{1}{1-n}}, \tag{13}$$

$$x=\xi+c_1t+\frac{2\alpha[\rho_{11}(\xi)]^n}{(2-n)\beta k}-\frac{2\alpha[\rho_{11}(\xi)]^n}{(2-n)\beta k}\left\{1-\frac{(1-n)\beta k}{[\rho_{11}(\xi)]^{1-n}}t\right\}^{\frac{2-n}{1-n}} \quad \text{at } n\neq 1;$$

$$\rho_{12}=\frac{\rho_1}{\rho_{11}(\xi)}=e^{-\beta kt},\quad x=\xi+c_1t+2\alpha\frac{1-e^{-\beta kt}}{\beta k}\rho_{11}(\xi) \quad \text{at } n=1; \tag{14}$$

for an Oldroyd viscoelastic liquid

$$\rho_1=\frac{\rho_{11}(\xi)}{1-\frac{\alpha_1^2}{\alpha_2^2}\frac{(1+\alpha_2\theta)}{(1+\alpha_1\theta)}}\left[\left(1+\frac{1+\alpha_2\theta}{\alpha_2^2\theta^2}\right)e^{\alpha_1 t}-\left(\frac{1+\alpha_2\theta}{\alpha_2^2\theta^2}+\frac{\alpha_1^2}{\alpha_2^2}\frac{1+\alpha_2\theta}{1+\alpha_1\theta}\right)e^{\alpha_2 t}\right], \tag{15}$$

$$x=\xi+\frac{c_1}{1-\beta k_1(\theta-\lambda)}t+\frac{2\alpha-\beta k_1(\theta-\lambda)\frac{c_1}{\rho_{00}}}{1-\beta k_1(\theta-\lambda)}\frac{1}{1-\frac{\alpha_1^2}{\alpha_2^2}\frac{(1+\alpha_2\theta)}{(1+\alpha_1\theta)}}$$

$$\left[\left(1+\frac{1+\alpha_2\theta}{\alpha_2^2\theta^2}\right)\frac{e^{\alpha_1 t}-1}{\alpha_1}+\left(\frac{1+\alpha_2\theta}{\alpha_2^2\theta^2}+\frac{\alpha_1^2}{\alpha_2^2}\frac{1+\alpha_2\theta}{1+\alpha_1\theta}\right)\frac{1-e^{\alpha_2 t}}{\alpha_2}\right]\rho_{11}(\xi)$$

for $(1+\beta k_1\lambda)^2>4\theta\beta k_1$;

$$\rho_1=\rho_{11}(\xi)\left[1-\frac{1+2\alpha_{10}\theta+2\alpha_{10}^2\theta^2+\alpha_{10}^3\theta^3}{2\alpha_{10}\theta^2+\alpha_{10}^2\theta^3}t\right]e^{\alpha_{10}t}, \tag{16}$$

$$x=\xi+\frac{c_1}{1-\beta k_1(\theta-\lambda)}t+\frac{2\alpha-\beta k_1(\theta-\lambda)\frac{c_1}{\rho_{00}}}{1-\beta k_1(\theta-\lambda)}\left\{\frac{e^{\alpha_{10}t}-1}{\alpha_{10}}-\frac{1+2\alpha_{10}\theta+2\alpha_{10}^2\theta^2+\alpha_{10}^3\theta^3}{2\alpha_{10}\theta^2+\alpha_{10}^2\theta^3}\frac{1}{\alpha_{10}^2}\left[e^{\alpha_{10}t}(\alpha_{10}t-1)+1\right]\right\}\rho_{11}(\xi)$$

for $(1+\beta k_1\lambda)^2=4\theta\beta k_1$;

$$\rho_1=\frac{\rho_{11}(\xi)}{\sin\varphi}e^{\alpha_{10}t}\sin(\alpha_{11}t+\varphi), \tag{17}$$

$$x=\xi+\frac{c_1}{1-\beta k_1(\theta-\lambda)}t+\frac{2\alpha-\beta k_1(\theta-\lambda)\frac{c_1}{\rho_{00}}}{1-\beta k_1(\theta-\lambda)}\frac{1}{(\alpha_{10}^2+\alpha_{11}^2)\sin\varphi}\times$$

$$+[\alpha_{10}e^{\alpha_{10}t}\sin(\alpha_{11}t+\varphi)-\alpha_{10}\sin\varphi-\alpha_{11}e^{\alpha_{10}t}\cos(\alpha_{11}t+\varphi)+\alpha_{11}\cos\varphi]\rho_{11}(\xi)$$

for $(1+\beta k_1\lambda)^2<4\theta\beta k_1$,

where

$$\alpha_{1/2}=\frac{-(1+\beta k_1\lambda)\pm\sqrt{(1+\beta k_1\lambda)^2-4\theta\beta k_1}}{2\theta},$$

$$\alpha_{10} = -\frac{1+\beta k_1\lambda}{2\theta}, \quad \alpha_{11} = \frac{\sqrt{4\theta\beta k_1 - (1+\beta k_1\lambda)^2}}{2\theta},$$

$$\varphi = \operatorname{arctg}\left\{\frac{\sqrt{4\theta\beta k_1 - (1+\beta k_1\lambda)^2}\,[1-\beta k_1(\theta-\lambda)]}{1-\beta k_1\theta + \beta^2 k_1^2\lambda(\lambda-\theta)}\right\}.$$

We will first analyze the features of wave propagation in a power-law liquid.

It is evident from Eqs. (13), (14) that at $n = 1$ (viscous liquid) and $n > 1$ (dilatant liquid) the initial density perturbation damps out, taking on a zero value as $t \to \infty$. At $n < 1$ (pseudoplastic liquid) the density value vanishes upon expiration of a finite time $t = T$, which is determined by the first equation of Eq. (13) and equal to $T = [\rho_{11}(\xi)]^{1-n}/(1-n)\beta k$.

It follows from the latter that in pseudoplastic liquids a perturbation wave propagates to a finite depth which is given by the expression

$$x - \xi = c_1\frac{[\rho_{11}(\xi)]^{1-n}}{(1-n)\beta k} + \frac{2\alpha\,[\rho_{11}(\xi)]^{n}}{(2-n)\,\beta k}.$$

This most important effect, caused by the nonlinearity of the dissipation law, explains certain phenomena in rheologically complex liquids. For example, it is impossible to start up pipelines containing pseudoplastic petroleums after a certain shutdown period.

From the second expressions of Eqs. (13), (14) it is evident that nonlinearity in the equation of state of the liquids leads to distortion of the wave profile. The possibility of wave upset may be determined by differentiating Eqs. (13), (14) with respect to the parameter ξ.

The approximate time and section in which upset may occur can be estimated in the following manner. Let the wave profile at the initial time have the form $\rho_{11}(\xi)$. At $\xi = 0$ $\rho_{11}(0) = \rho_{10}$, while at $\xi = \xi_1$ $\rho_{11}(\xi_1) = 0$, where ξ_1 is the width of the wave at $t = 0$. It is evident from Eq. (11) that the speed of the wave corresponding to $\xi = 0$ is equal to

$$c_1 + 2\alpha\,\rho_{10}\left\{1 - \frac{(1-n)\,\beta k}{\rho_{10}^{1-n}}\,t\right\}^{\frac{1}{1-n}}$$

at $n \neq 1$ and $c_1 + 2\alpha\rho_{10}e^{-\beta kt}$ at $n = 1$, while at $\xi = \xi_1$ it is equal to c_1. Since $\alpha > 0$, then $c_1 + 2\alpha\rho_1 > c_1$.

Then the wave upset time can be determined from the expressions

$$T_0\left\{1 - \frac{(1-n)\,\beta k}{\rho_{10}^{1-n}}\,T_0\right\}^{\frac{1}{1-n}} = \frac{\xi}{2\alpha\rho_{10}} \quad \text{for} \quad n \neq 1,$$

$$T_0 e^{-\beta k T_0} = \frac{\xi}{2\alpha\rho_{10}} \quad \text{for} \quad n = 1.$$

Substituting the values found for T_0 in the second expression of Eq. (13) and (14), we determine the sections in which upset of the perturbation wave occurs. For pseudoplastic liquids $(n < 1)$ wave upset can occur under the condition $T_0 < T$. In the opposite case the perturbation wave damps out before upsetting.

If $\alpha = 0$ (pressure depends linearly on density) then each point of the wave moves with an identical velocity c_1 and upset cannot occur. Figure 1 shows qualitative curves of the change in density with time and a characteristic diagram for nonlinear perturbation waves in power-law rheologically complex liquids with step-like density change.

Inasmuch as at the initial time in the section $x = 0$ there is a maximum density value, the wave velocity is maximum independent of the parameter n, and equal to $c_1 + 2\alpha\rho_{10}$. In Fig. 1 this velocity is characterized by the tangent of the angle φ at the points $t = 0$, $x = 0$.

Upon propagation of the perturbation wave the density value ρ_1 tends to zero. Therefore, the wave velocity tends to the value c_1, characterized by the tangent of the angle ψ (Fig. 1) independent of the nonlinearity parameter n. At $\alpha = 0$ the wave velocity c_1 is constant and characterized by the angle ψ, passing through the origin.

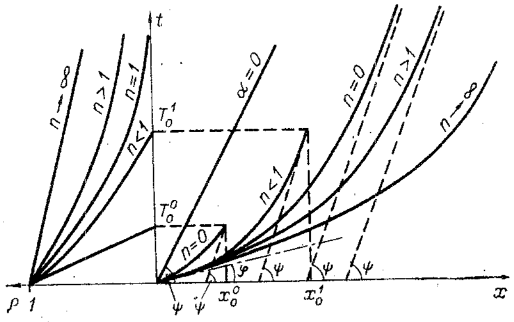

Fig. 1

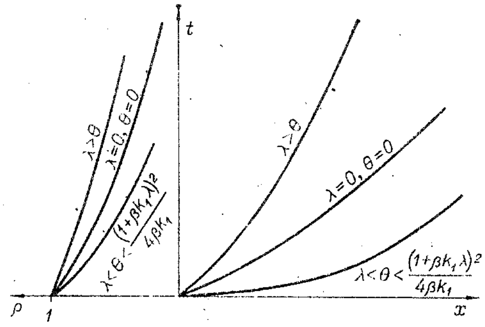

Fig. 2

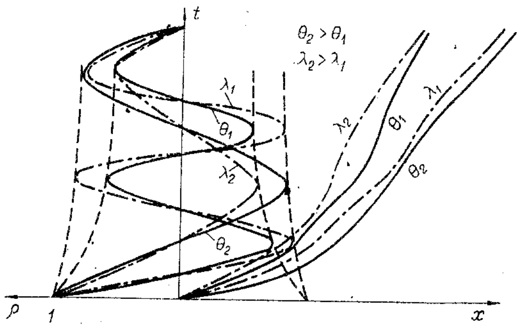

Fig. 3

We will now analyze the features of perturbation wave propagation in Oldryod viscoelastic liquids. It is evident from Eqs. (15) and (17) that under the condition $\theta \leq (1 + \beta k_1 \lambda)^2/(4\beta k_1)$ the initial density perturbation decreases monotonically, while in the case $\lambda < \theta \leq (1 + \beta k_1\lambda)^2/(4\beta k_1)$ the density value attenuates more rapidly than in liquids with parameters $\lambda = 0$, $\theta = 0$ (viscous liquid) and $\lambda > \theta$.

For viscoelastic liquids upon fulfillment of the condition $\theta > (1 + \beta k_1\lambda)^2/4\beta k_1)$ the initial density perturbation oscillates with a frequency $\sqrt{4\theta\beta k_1 - (1) + \beta k_1\lambda)^2}/2\theta$, which decays exponentially with the parameter $-(1 + \beta k_1\lambda)/2\theta$. It is evident that with increase in θ the frequency and damping parameter decrease, while increase in λ increases the value of the damping parameter and decreases the oscillation frequency.

We may note the following regarding the velocity of perturbation wave propagation. The presence in the liquid of a relaxation time θ increases the wave propagation speed and decreases the effect of nonlinearities on perturbation wave propagation as $\theta \to 2\alpha\rho_{oo}/(\beta k_1 c_1) + \lambda$, while the presnece of a retardation time λ decreases the wave propagation velocity and increases the effect of nonlinearities. Under the condition $2\alpha - \beta k_1(\theta - \lambda)c_1\,\rho_{oo} = 0$ the perturbation wave propagates as in a linear viscoelastic liquid.

Characteristic curves of change in density with time and perturbation wave characteristic diagrams in a viscoelastic liquid with step-like density change, i.e., with $t = 0$, $x = 0$, $\rho_{11}(\xi) \equiv \rho_{10}$ are shown in Figs. 2, 3.

Analysis of the second expression of Eq. (12) with consideration of the first expressions of Eqs. (15) and (16) shows that with increase in relaxation time θ and decrease in retardation time λ the upset time and section increase. Numerical values of these quantities can be estimated as was done above.

The results obtained in the present study may be used to solve concrete engineering problems involving optimization of transient flow regimes of rheologically complex liquids in petroleum, chemical technology, and other branches of industry.

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FLOW DETACHMENT FROM THE LEADING EDGE OF A PROFILE AND THE EFFECT OF ACOUSTIC PERTURBATIONS

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The study of the phenomenon of detachment of a flow has long attracted the interest of researchers because of the wide extent to which detachment flows are found, and their major role in flow structure formation. It is known that two different flow regimes may exist after detachment [1]. In some cases the initial boundary layer passes above the region of recirculating liquid and then again attaches to the body at some point down the flow, separating "bubbles" of recirculating liquid. In other cases liquid from the boundary layer does not reattach to the body, but travels down the flow, mixing with the recirculating liquid and forming a wake. In this case for a profile oriented at a large angle of attack, detachment encompasses the entire upper surface.

The flow regimes described above determine the type of detachment. The detachment may be "localized," as in the first case, or may include the entire surface, as in the second.

The first type of detachment was realized in [2, 3]. In this case, a small "localized" detachment was formed in the midpart of the wing profile. It was shown in [2] that natural perturbations developing in the detachment region may lead to significant readjustment of the flow structure in this region. It was found in [3] that in the region of unfavorable pressure gradient acoustic perturbations are transformed to turbulent boundary perturbations (Tollmien–Schlichting waves), which propagate down the flow, which also have a strong effect on the structure of the laminar flow in the boundary layer and may lead to elimination of the detachment as in the case where perturbations are introduced into the boundary layer by a vibrating ribbon.

The goal of the present study is to generate a detachment encompassing the entire upper surface of the profile, i.e., a detachment of the second type, and to study its structure and the effect thereon of acoustical perturbations.

The experiments were performed in a T-324 low turbulence aerodynamic tube at the Siberian Branch of the Academy of Sciences of the USSR [4]. The test chamber dimensions were 1 × 1 m with length of 4 m. Flow detachment was studied with a symmetrical Zhukovskii airfoil 1 with chord of 292 mm and span of 1 m, located at an attack angle of 11° at a distance of 1.0 m from the beginning of the chamber. A diagram of the experimental setup is shown in Fig. 1. A loudspeaker 2 was installed in the tube diffusor to excite acoustical oscillations in the flow region to be studied. The loudspeaker was driven by a GZ-34 audio generator. A microphone located in the direct vicinity of the model and a PSI-202 precision pulse noise meter were used to measure the integral over spectrum of the sound intensity, which was maintained at $A_s = 90$ dB in the present experiments (background sound intensity 80 dB).

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